for the problem. Away from the instability speed, the response rms is generally of the order of $10x^{-2}$. As shown in Figs. 1–3, the sensitivity is of order 1. Therefore, the sensitivities are approximately 3 orders of magnitude larger than the actual response. In addition, the sensitivity of the gust response can vary in sign over the range of nondimensional airspeeds examined here. Consequently, if one were to try to use aeroelastic tailoring for minimizing the gust response, extreme care would have to be taken in approach. In particular, if an optimization scheme were to be used, it would have to be fairly robust to allow for wide variations in the objective function (the gust response). In addition, one could use these sensitivities as guides for a preliminary examination (parameter studies or trade studies) of designing the wing for the gust response.

This Note has laid the groundwork for a better understanding of how the rms gust response and its sensitivity vary with airspeed and structural parameters. It has shown that the sensitivity of the gust response is much larger than the actual response by approximately 3 orders of magnitude. Consequently, the opportunity for aeroelastic tailoring for gust response has been demonstrated, but care must be taken when performing the tailoring.

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Thrust Offset Effect on Longitudinal Dynamic Stability

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Nomenclature

A	=	aspect ratio
C_D	=	drag coefficient
$C_{D_{\min}}$	=	minimum or parasitic drag coefficient
C_L	=	lift coefficient
C_{L_α}	=	$\partial C_L/\partial\alpha$
C_m	=	pitching moment coefficient
C_{m_α}	=	$\partial C_m/\partial\alpha$
$\bar{c}$	=	mean aerodynamic chord
D	=	drag
e	=	efficiency factor for wing-induced drag
g	=	acceleration caused by gravity
I_Y	=	moment of inertia of aircraft about y axis
K_n	=	static margin, $-C_{m_\alpha}/C_{L_\alpha}$
L	=	lift
M	=	pitching moment about the c.g.
$M_{q,u,w,\dot{w}}$	=	$(\partial M/\partial q)/I_y$, $(\partial M/\partial u)/I_y$, $(\partial M/\partial w)/I_y$, $(\partial M/\partial \dot{w})/I_y$
m	=	aircraft mass
$Oxyz$	=	axes fixed in aircraft
q	=	angular velocity in pitch
S	=	wing area
T	=	thrust
t	=	time
u	=	velocity component in x direction
u_0	=	reference flight velocity
W	=	aircraft weight
w	=	velocity component in z direction
X	=	force component along $0x$
$X_{u,w}$	=	$(\partial X/\partial u)/m$, $(\partial X/\partial w)/m$
Z	=	force component along $0z$
$Z_{u,w}$	=	$(\partial Z/\partial u)/m$, $(\partial Z/\partial w)/m$
z_{TH}	=	perpendicular distance of thrust line below aircraft c.g.
α	=	angle of attack
Δh_n	=	neutral point shift because of thrust line offset determined from trim-slope criterion
Δu	=	perturbation of aircraft velocity component in x direction
Δw	=	perturbation of aircraft velocity component in z direction
$\Delta\theta$	=	pitch deviation angle
λ	=	root of stability equation
ρ	=	density of air

Subscript

0	=	reference flight

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Introduction

THRUST lines of action that pass above or below the c.g. affect both the trim and stability of an aircraft. Examples include flying boats with high-mounted engines or transport aircraft with engines mounted on pylons below a low wing. Direct thrust effects on trim are relatively straightforward to evaluate, but stability changes with power require more consideration. The definition of the neutral point has caused some confusion. Standard flight tests rely on elevator deflections measured at different trimmed flight speeds for a range of c.g. positions. The neutral point is then defined as the c.g. position at which the gradient of the elevator deflection to trim vs lift coefficient is zero. For a constant-thrust jet engine this leads to a neutral point shift Δh_n caused by thrust offset given by Solies[1] as

$$\Delta h_n = -(T/W)(z_{TH}/\bar{c}) \tag{1}$$

A low-mounted engine with $z_{TH} > 0$ is considered as destabilizing, whereas a high-mounted engine with $z_{TH} < 0$ is stabilizing. However, Etkin[2] states that the application of the trim-slope criterion can be misleading as to stability and defines the neutral point on the basis of the pitch stability term $C_{m_\alpha}(=\partial C_m/\partial\alpha)$. A similar view is expressed by Solies.[1]

To obtain a clearer understanding of thrust offset effects it is necessary to consider dynamic stability by solving the usual linearized, longitudinal equations of motion.

Equations of Motion

For initial horizontal flight and neglecting the aerodynamic derivatives $Z_{\dot{w}}$ and Z_q, Nelson[3] gives the following linearized, longitudinal equations of motion:

$$\begin{gathered}\left(\frac{d}{dt} - X_u\right)\Delta u - X_w\Delta w + g\Delta\theta = 0\\ -Z_u\Delta u + \left(\frac{d}{dt} - Z_w\right)\Delta w - u_0\frac{d}{dt}\Delta\theta = 0\\ -M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = 0\end{gathered} \tag{2}$$

where the force derivatives are divided by m and the moment derivatives are divided by I_Y. Solution of the preceding equations for a stable aircraft typically gives two modes, namely, the phugoid and short period oscillations.

The usual approximation to the short period mode is made by assuming $\Delta u = 0$ and dropping the X-force equation with the resulting set of two equations independent of the thrust offset effect contained within the M_u derivative. The usual boundary condition for neutral stability of the short-period mode applies viz. that the constant term in the characteristic quadratic equation is zero giving $\lambda = 0$ as a root. The constant term $(M_q Z_w - u_0 M_w)$ is proportional to the maneuver margin based on pitch changes only.

In the case of the phugoid oscillation, the pitching moment equation reduces to

$$-M_u \Delta u - M_w \Delta w = 0 \tag{3}$$

with $\Delta w = 0$, i.e., constant angle of attack, if $M_u = 0$. The characteristic equation in λ is given by

$$\begin{vmatrix} (\lambda - X_u) & -X_w & g \\ -Z_u & (\lambda - Z_w) & -u_0\lambda \\ -M_u & -M_w & 0 \end{vmatrix} = 0$$

i.e.,

$$\lambda^2 + \left[-X_u + \frac{M_u}{M_w}\left(X_w - \frac{g}{u_0} \right) \right] \lambda + \frac{g}{u_0}\left(-Z_u + Z_w \frac{M_u}{M_w} \right) = 0 \tag{4}$$

Neglecting compressibility and aeroelastic effects and taking the constant thrust jet aircraft as an example, the aerodynamic derivatives which appear in Eq. (4) are given by

$$X_u = -\rho u_0 S C_{D_0}/m \tag{5a}$$

$$Z_u = -\rho u_0 S C_{L_0}/m \tag{5b}$$

$$X_w = \tfrac{1}{2}\rho u_0 S[C_{L_0} - (2C_{L_0}/\pi A e)C_{L_\alpha}]/m \tag{5c}$$

$$Z_w = -\tfrac{1}{2}\rho u_0 S C_{L_\alpha}/m \tag{5d}$$

$$M_w = \tfrac{1}{2}\rho u_0 S \bar{c} C_{m_\alpha}/I_y \tag{5e}$$

C_D is assumed to be given by the usual drag polar $C_D = C_{D_{\min}} + C_L^2/\pi A e$.

The pitching moment as a result of forward speed derivative M_u is derived by referring to Fig. 1, where the pitching moment because of thrust in the reference flight is balanced by an aerodynamic control moment. As shown in the Fig. 1, shifting the thrust line to a position below the aircraft c.g. requires a down-elevator movement to trim the aircraft. There is also an aerodynamic moment about the aircraft c.g. that varies with the aircraft angle of attack or lift coefficient and is balanced by the elevator in the reference flight. This latter moment balance remains if the aircraft forward speed changes, provided there is no change in the aircraft angle of attack. Forward speed changes do affect the balance of the thrust moment because the thrust moment is constant, whereas the aerodynamic control moment varies with the square of the forward speed. Because constant angle of attack is implied in the derivation of the partial derivative M_u, the pitching moment about the c.g. can be written in terms of the forward speed u as the sum of the thrust moment and aerodynamic control moment. The pitching moment is given by

$$M = T z_{TH} - T z_{TH}(u/u_0)^2 \tag{6}$$

In the reference flight $M = 0$ and $T = D_0$. Differentiation with respect to u gives

$$M_u = \frac{1}{I_y}\left(\frac{\partial M}{\partial u} \right)_0 = -\frac{2 T z_{TH}}{I_y u_0} \tag{7}$$

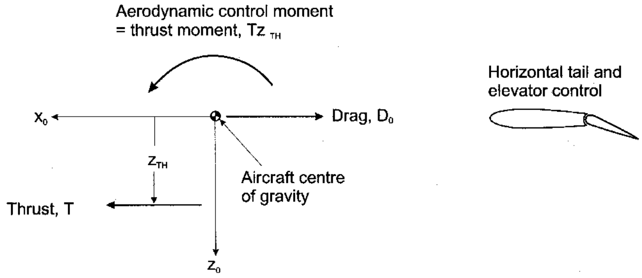

Fig. 1 Pitching moment balance in reference flight.

The thrust moment therefore makes no direct contribution to M_u, which results from the variation of the aerodynamic control moment with speed.

Substituting the aerodynamic derivative expressions and simplifying the characteristic equation using $L_0 = \frac{1}{2}\rho u_0^2 S C_{L_0} = W$ gives

$$\lambda^2 + \frac{2g}{u_0}\left[\frac{C_{D_0}}{C_{L_0}} + \frac{2C_{L_0}}{\pi A e}\frac{C_{L_\alpha}}{C_{m_\alpha}}\left(\frac{z_{TH}}{\bar{c}}\right)\left(\frac{T}{W}\right) \right] + \frac{2g^2}{u_0^2}\left[1 + \frac{C_{L_\alpha}}{C_{m_\alpha}}\left(\frac{z_{TH}}{\bar{c}}\right)\left(\frac{T}{W}\right) \right] = 0 \tag{8}$$

which can be rewritten as

$$\lambda^2 + \frac{2g}{u_0}\left(\frac{C_{D_0}}{C_{L_0}} + \frac{2C_{L_0}}{\pi A e}\frac{\Delta h_n}{K_n} \right) + \frac{2g^2}{u_0^2}\left(1 + \frac{\Delta h_n}{K_n} \right) = 0 \tag{9}$$

where Δh_n is the neutral point shift caused by the thrust offset given by Eq. (1), and $K_n(= -C_{m_\alpha}/C_{L_\alpha})$ is the static margin based on pitch changes only. In the case of a statically stable aircraft ($K_n > 0$), the phugoid damping and frequency are enhanced by a high-thrust line ($z_{TH}/\bar{c} < 0$ and $\Delta h_n > 0$). A low-thrust line reduces damping with a neutrally stable root corresponding to $(1 + \Delta h_n/K_n) = 0$, i.e., $\Delta h_n = -K_n$. At this condition the other root is stable if $C_{D_0} > 2C_0^2/\pi A e$, i.e., the jet aircraft is flying above the minimum drag speed.

Conclusions

Thrust offset indirectly affects dynamic stability through its influence on the aerodynamic derivative M_u. To a first approximation high- or low-thrust lines do not affect the longitudinal short-period oscillation with a positive maneuver margin required for stability. The phugoid oscillation, however, is adversely affected by a low-thrust line with the phugoid damping reduced. In the case of a constant thrust jet aircraft flying above the minimum drag speed and neglecting compressibility effects, the condition for neutral stability has been derived. This condition states that neutral stability occurs at the point where the neutral point shift because of a low-thrust line and determined from the trim-slope criterion is equal to the static margin based on pitch changes only. A high-thrust line has the opposite effect on the phugoid, increasing its damping and frequency.

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